each locality it alters the level of the surface by its weight, and the direction of the vertical by its horizontal attraction; for material that is homogeneous elastically, the ratio of these two effects is in all cases the same, being $[(1-\sigma)/\mu](g/\gamma)(g/2\pi)$, where μ is the rigidity of the material and σ its ratio of lateral contraction to elongation, g is gravity, and g/γ is the mass of the earth divided by the square of its radius.* Thus, for steel, taking $\sigma = 0{\cdot}27$, $\mu = 8 \times 10^{11}$, the deflection of the surface is always and everywhere 2·1 times the deflection of the vertical, the total effect being their sum. For glass the ratio is 6·7.

The complete results for various distributions of load can be written down at once from known expressions for cases of attraction: for example, the case of elliptic loading (over any elliptic area), which is considered at the end of the paper.—J. L.]

Historical Note on a Relation between the Gravitational Attraction Exercised and the Elastic Depression Caused by Load on the Plane Surface of an Isotropic Elastic Solid.

By C. Chree, F.R.S.

(Received August 15, 1917.)

So far as is known, the result to which the present note refers was first given in the special form which it assumes for an incompressible material in a note written by Sir G. H. Darwin entitled, "On Variations in the Vertical due to Elasticity of the Earth's Surface," included in the British Association Report for 1882 as an Appendix to the Report of a Committee appointed for the "Measurement of the Lunar Disturbance of Gravity." Sir G. H. Darwin and Lord Kelvin (then Sir William Thomson) were members of this Committee.

On p. 108 (*loc. cit.*) Darwin writes, "Before proceeding further I shall prove a very remarkable relation between the slope of the surface of an elastic horizontal plane and the deflection of the plumb line caused by the direct attraction of the weight producing that slope. This relation was pointed out to me by Sir William Thomson, when I told him of the investigation on which

* Since writing the above I have found that this general relation was stated by Dr. C. Chree in 1897 ('Phil. Mag.,' vol. 43, p. 177), with some particular applications to cases of rectangular loaded areas; like results had previously been given by Sir G. Darwin for the special case of a system of parallel ridges.

I was engaged; but I am alone responsible for the proof as here given. He writes that he finds that it is not confined simply to the case where the solid is incompressible, but in this paper it will only be proved for that case."

To understand the result it is necessary to mention that Darwin assumed that so far as the phenomena in question were concerned, the earth's surface might be regarded as plane, except for the presence of mountains and valleys, which he regarded as equivalent to a load varying as $\cos z/b$. Darwin used the terms "deflection" and "slope"—which I shall presently distinguish as ψ_2 and ψ_1—to indicate respectively the change in the direction of gravity due to the direct attraction, and the slope introduced by the elastic depression. He also employed ν for the rigidity of the incompressible material, a for the radius and δ for the mean density of the earth, and g for the acceleration of gravity.

After obtaining certain formulæ he proceeds, "Therefore deflection bears to slope the same ratio as ν/g to $\frac{1}{3}a\delta$. This ratio is independent of the wave-length $2\pi b$ of the undulating surface, of the position of the origin, and of the azimuth in the plane of the line normal to the ridges and valleys. Therefore the proposition is true of any combination whatever of harmonic undulations, and as any inequality may be built up of harmonic undulations, it is generally true of inequalities of any shape whatever."

It would appear that it was Lord Kelvin who first noticed the relation. Whether he had arrived at it quite independently of and prior to Darwin's investigation, having actually solved the problem presented by compressible isotropic material and reached a definite formula, or whether he simply noticed that the result would follow for an incompressible material from Darwin's formulæ, and inferred by general reasoning that it was not confined to incompressible material, it is impossible to say. If he did reach a definite formula, apparently he did not communicate it to Sir G. H. Darwin. The latter's remarks are not explicit, but they certainly suggest that he was unaware that the rigidity is not the only elastic constant involved when the material is compressible. In his numerical applications he employs for examples the rigidities of glass and steel, without any explicit warning that in actual glass and steel the results would have been widely different.

The earliest publication of the result for ordinary compressible isotropic material was, I believe, in a paper "Applications of Physics and Mathematics to Seismology," read before the Physical Society of London in December, 1896

Equation (9), p. 178, *loc. cit.*, gives explicitly $\psi_1/\psi_2 = (1-\eta)g^2/2\pi n\gamma$, where γ is the gravitational constant, while n and η denote the rigidity and Poisson's ratio of the material. If we put $\eta = \frac{1}{2}$, the appropriate value for incompressible material, and write $g/\frac{4}{3}\pi\delta a$ for γ, and ν for n, we have Darwin's result. If, however, we take $\frac{1}{4}$ instead of $\frac{1}{2}$ for η, keeping the rigidity

unaltered, we raise the value of ψ_1/ψ_2 by 50 per cent., so the question of the compressibility of the material is not unimportant. The above result agrees with Sir Joseph Larmor's (p. 14, *supra*) and with that obtained by combining Terazawa's equations (6) and (7).*

I was not aware of the existence of Sir G. H. Darwin's paper until long after my paper was published, and did not notice the reference in it to Lord Kelvin until the publication of Terazawa's paper led me to restudy the problem.

Experiments on Tribo-Electricity. I.—*The Tribo-Electric Series.*

By P. E. Shaw, B.A., D.Sc., University College, Nottingham.

(Communicated by Prof. E. H. Barton, F.R.S. Received May 24, 1917.)

CONTENTS.

The term "Tribo-electricity" is used in O. D. Chwolson's 'Traité de Physique' (1). This term is a convenient equivalent for frictional electricity (τρῐβή = a rubbing).

I.—*Historical.*

Little exact knowledge of tribo-electricity has yet been accumulated, and this subject has certainly not been raised to the dignity of a quantitative science. In the present paper will be found an account of experiments in which the conditions of the solid bodies rubbed together have been greatly varied. Thus, temperature has been changed both before and during friction; the surfaces used have been rubbed with considerable pressure while hot; they have, wherever possible, been rubbed together when flexed; and they have been prepared before rubbing by being ground and polished in various ways. Much information has thus come to light as to the electrical surface conditions of a variety of solids.

* 'Phil. Trans.,' A, vol. 217, p. 37.